SM3 HW 12.4 Margin of Error

OBJECTIVES:

- Make inferences and justify conclusions from sample surveys, experiments and observational studies
- Use data from a sample survey to estimate a population mean or proportion.
- Develop a margin of error through the use of simulation models for random sampling.

VOCABULARY:

- **Inference** in statistics is using the information taken from a smaller sample to make conclusions about the entire population of interest
- **Margin of error** accounts for the unavoidable variation in results if the study or simulation were to be conducted multiple times under the same conditions. It gives us an idea of the interval which we expect our population parameter to be with in.
- A **confidence interval** provides a range of plausible values for a population parameter. It can be found by using the sample statistic and the margin of error for the confidence level desired
  [sample statistic $\pm$ margin of error]
- The **confidence level** is how likely it is that the actual population parameter falls within the confidence interval that was calculated. It is used to create the margin of error. The higher the confidence level the wider the margin of error will be, a low confidence level would create a smaller (narrower) margin of error.

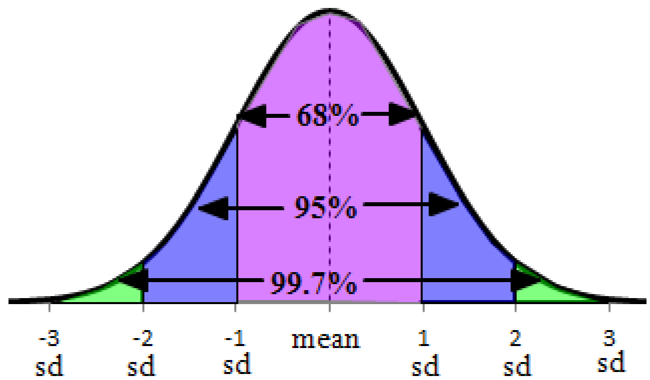
For example, if I wanted to guess what the average percentage on the unit test will be with 100% confidence level I could guess an interval like 0% to 100%. I am sure that the class average percentage is in the interval from 0% to 100% but that guess is not very helpful and doesn't tell me much about how well the classes will do on that test. The narrower my guess becomes, the less confident I am that I actually captured the average percentage for all my classes. I could guess an average of 80% - 82% which is a very precise guess but I have a high likelihood that the actual percentage isn't in that interval. We want to balance between a low margin of error and a high level of confidence. Often a confidence level of 95% is used.

When we use a simulation to model an event, it is only an approximation of the population parameter. If we were to run the simulation numerous times, each result would be slightly different. However, it is possible to give an interval that the population parameter falls within by finding a margin of error.

A margin of error is not a mistake; rather it refers to the expected range of variation in a survey or simulation if it were to be conducted multiple times under the same procedures. The margin of error is based on the sample size and the confidence level desired. The interval that includes the margin of error is called the **confidence interval** and is usually computed at a **95% confidence level**. A confidence level of 95% means that you can be 95% certain that the actual population parameter falls within the confidence interval.

For example, the student body officers at your school conducted a survey to determine whether or not a majority of the students dislike the music played in the hall during class changes. They randomly interviewed students and determined that 52% of the students disliked the music with a margin of error of 3% that was calculated at a 95% confidence level. Can the student body officers say for certain that over half of the student body dislikes the music? The confidence interval for this situation is or 49% to 55%. The confidence level of 95% allows us to say that we are 95% confident that the true percentage is between 49% and 55%. However, it is plausible that the percentage of students who dislike the music is less than 50%. Therefore, it is probably not wise to approach the administration about changing the music just yet.

When calculating the means from several trials of a simulation, the results are normally distributed. Recall that in a normal distribution 68% of the data falls within 1 standard deviation of the mean, 95% of the data falls within two standard deviations of the mean, and 99.7% of data falls within three standard deviations of the mean. We can use the fact that **95% of the data is within 2 standard deviations** of the mean to find a margin of error with a 95% confidence level.

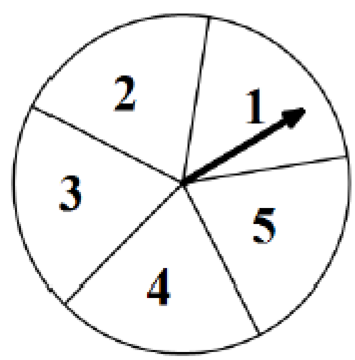


Calculating the Confidence Interval for a 95% Confidence Interval:
The following formulas can be used to approximate the margin of error with a **95% confidence level**:

1. For sample **PROPORTIONS:** The 95% confidence interval can be approximated by: $Sample\ Statistic\ \pm Margin\ of\ Error = \hat{p} \pm 2 \cdot \sigma_{\hat{p}}$ where $\hat{p}$ is the sample proportion and $\sigma_{\hat{p}}$ is standard deviation of $\hat{p}$
2. For sample **MEANS:** then the margin of error can be approximated by: $Sample\ Statistic\ \pm Margin\ of\ Error = \bar{x}\ \pm\ 2 \cdot \sigma_{\bar{x}}$, where $\bar{x}$ is the sample mean, $\sigma_{\bar{x}}$ is the standard deviation of $\bar{x}$

Example 1: A spinner like the one shown was spun 30 times and the number it landed on was recorded as shown below.

1	2	3	4	5
𝍸 |	||||	𝍸 |	𝍸	𝍸 ||||



For each situation, find the sample proportion, the margin of error for a 95% confidence level, the 95% confidence interval for the population proportion, and determine if the theoretical proportion would be within the confidence interval found.

a. **the probability of the spinner landing on 2**

The sample proportion is $\hat{p} = \frac{4}{30} \approx 0.133$ the $\sigma_{\hat{p}} = 0.062$

The 95% Confidence Interval would be:

$0.133\ \pm 2 \cdot (0.062) = 0.133\ \pm 0.124$

$$0.133 - 0.124 = 0.009 \quad and \quad 0.133 + 0.124 = .257$$

The 95% confidence interval is from 0.009 to 0.257

b. **the probability of the spinner landing on 5**

The sample proportion is $\hat{p} = \frac{9}{30} = 0.3$ so the with $\sigma_{\hat{p}} = 0.0836$ So the 95% Confidence Interval would be:

$0.3\ \pm\ 2 \cdot (0.0836) = 0.3\ \pm 0.167$

$0.3 - 0.167 = 0.133 \quad and \quad 0.3 + 0.167 = 0.467$ The 95% confidence interval is from 0.133 to 0.467

Example 2:

A fast food restaurant manager wanted to determine the wait times for customers in line. He timed 30 customers chosen at random and found that the sample mean was $\bar{x} = 5.7\ minutes$ and the st. dev. was $\sigma_{\bar{x}} = 0.4$

a. Approximate the margin of error for a 95% confidence level and round to the nearest tenth.

$$margin\ of\ error = 2 \cdot \frac{s}{\sqrt{n}} = 2 \cdot \frac{2.2}{\sqrt{30}} \approx 0.8$$

b. Find the 95% confidence interval.

$$5.7 \pm 0.8 \qquad 5.7 - 0.8 = 4.9 \qquad 5.7 + 0.8 = 6.5$$

The confidence interval is from 4.9 minutes to 6.5 minutes

d. Interpret the meaning of the interval in terms of wait times for customers.

We are 95% confident that the actual average wait time for a customer at the fast food restaurant is between 4.9 minutes and 6.5 minutes.

12.5 EXERCISES **NAME**___________________________________

1. A spinner like the one shown was spun 40 times and the number it landed on was recorded as shown below.

<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td></tr>
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For each situation, find the sample proportion, the margin of error for a 95% confidence level, the 95% confidence interval for the population proportion, and determine if the theoretical proportion of p=0.25 would be within the confidence interval found.

a. the probability of the spinner landing on 1 if the standard deviation of landing on a 1 is $\sigma_{\hat{p}} = 0.066$

$\rho =$ Margin of Error= Confidence Interval:

b. the probability of the spinner landing on 2

$\rho =$ Margin of Error= Confidence Interval:

c. the probability of the spinner landing on 4

$\rho =$ Margin of Error= Confidence Interval:

2. A consumer research group tested battery life of 36 randomly chosen cell phones to establish the likely battery life for the population of the newest model of iPhones. The sample mean is $\bar{x} = 68.6$ hours and the standard deviation of the mean is $\sigma_{\bar{x}} = 1.77$ minutes

   a. Find the margin of error for a 95% confidence level and round to the nearest tenth.

   b. Find the 95% confidence interval.

   c. Interpret the meaning of the interval in terms of battery life for this type of cell phone.

   d. Apple claims the new iPhone will have an average battery life of 75 hours, does the confidence interval found in part c support or refute Apple's claim?

   e. What factors may lead to an inaccurate confidence interval?

   f. What could be done to decrease the chance of these factors?

3. In a poll of 650 likely voters, 338 indicated that they planned to vote for a particular candidate.

   The $\sigma_{\widehat{p}} = 0.0195$

   a. Find the sample proportion.

   b. Approximate the margin of error for a 95% confidence level.

   c. Find the 95% confidence interval.

   d. Interpret the meaning of the interval in terms of the election.

   e. If there are only two candidates in the election, based on the confidence interval is it plausible that the candidate could win the election? Explain your reasoning.

   f. Is it plausible that the candidate could lose the election? Explain your reasoning

4. A sample of 100 patients suffering from high blood sugar were given a new medication to see if it lower their long term blood sugar levels called A1C. Each patient was tested for their A1C level at the beginning of the trial and then after 6 months of taking the new medication each patient was again tested for their A1C. The experimenters then calculated the difference in their A1C levels by finding Start A1C – End A1C. So a positive difference indicates the A1C lowered over the 6 months and a negative difference indicates the A1C increased over the 6 month trial.

   The average A1C difference for the 100 patients was $\bar{x} = 2.3$ and the standard deviation of $\bar{x}$ is $\sigma_{\bar{x}} = 1.7$

   a. Approximate the margin of error for a 95% confidence interval.

   b. Find the 95% confidence interval.

   c. Interpret the meaning of the interval in terms of the medication experiment.

   d. Does it appear that the medication helps lower A1C levels? Explain your reasoning.

   e. Does it appear that the medication is guaranteed to lower A1C levels? Explain your reasoning.

   f. If I decided to only add or subtract one standard deviation to $\bar{x}$ instead of two, what would my confidence level be? Would the interval be wider or narrower than the 95% confidence interval?

   g. How many standard deviations would I need to have a 99.7% confidence level?

   h. Calculate the 99.7% confidence interval for this problem.